Majorana modes in 3D topological insulators with warped surface state

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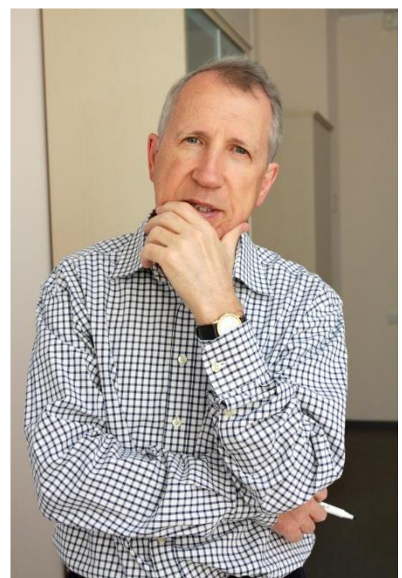


DIPC

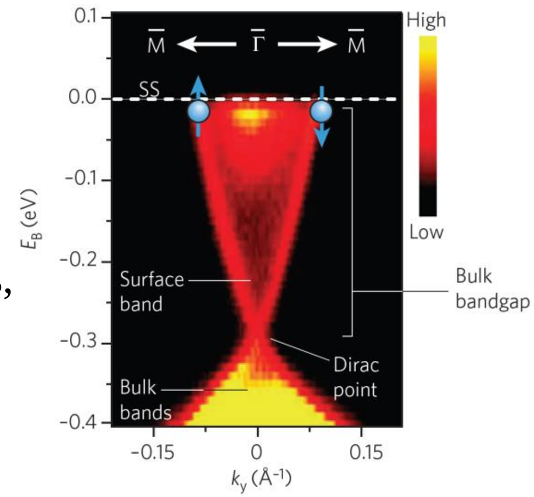
Topological insulator in a nutshell

…a new state of matter that has been predicted and discovered!

- Bulk is insulating; edge (2D)/ surface (3D) a very good conductor.
- Important ingredient: spin-orbit coupling:

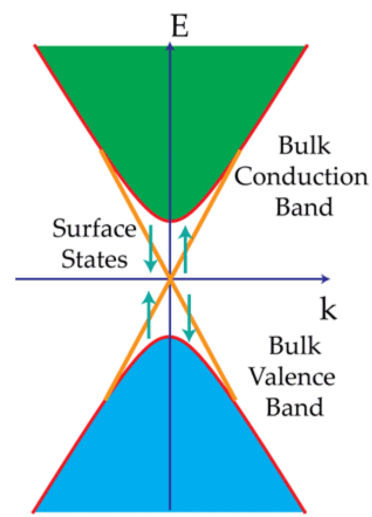
opposite force for opposite spins.

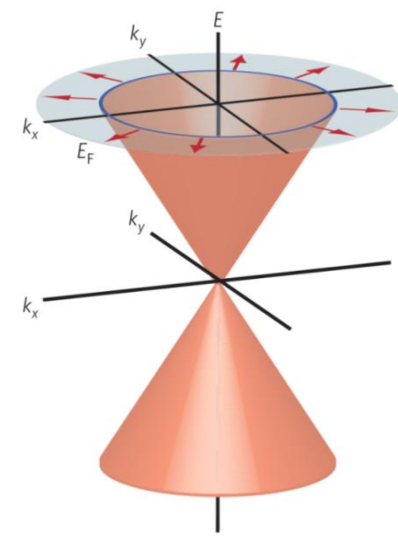
- Topological invariant is insensitive to any continuous deformation of Hamiltonian (topological protection): disorder, geometry, weak interactions, etc…



Examples:

- 2D: HgTe/CdTe; 3D: Bi_2Se_3, Bi_2Te_3 Sb_2Te_3, $TlBiSe_2$, Bi_2Te_2Se.





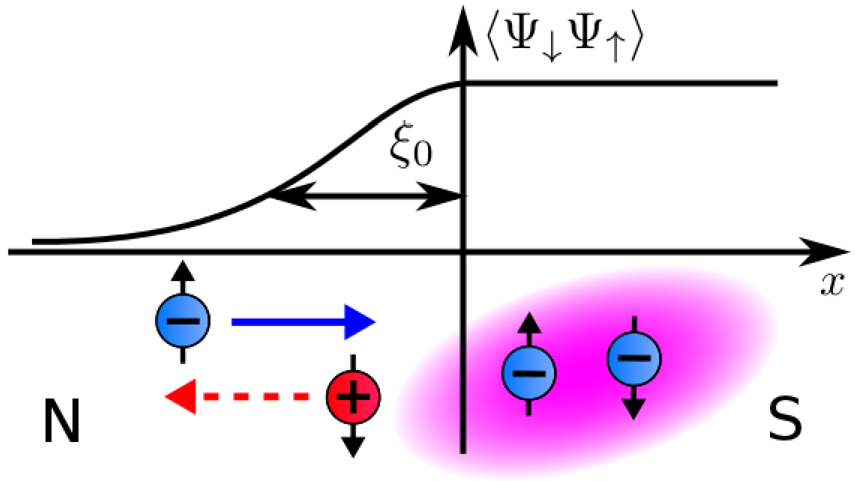
2D

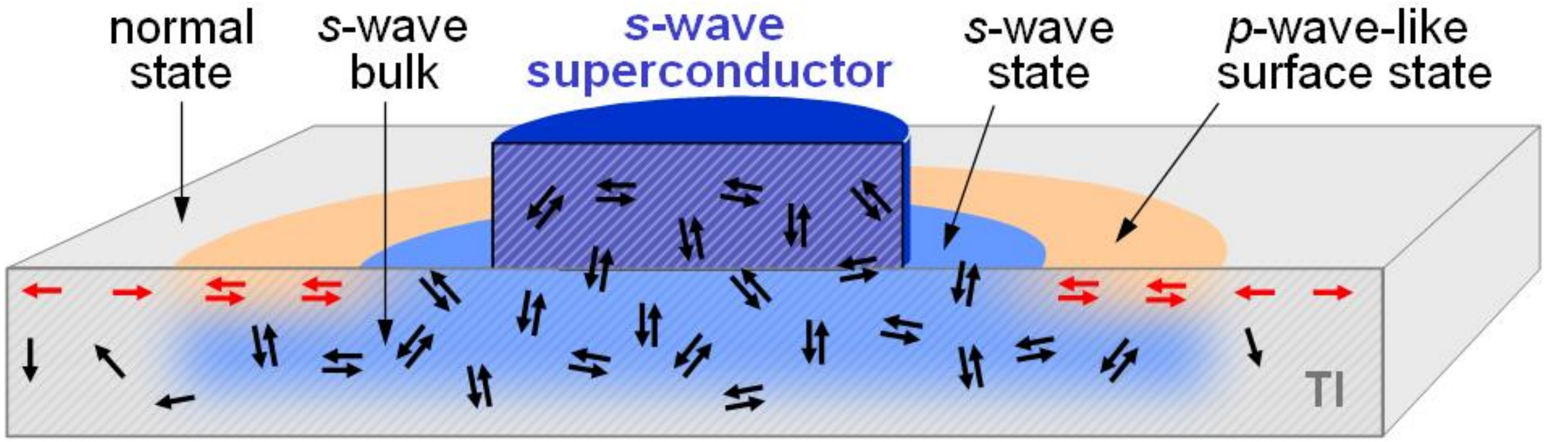
Theo1: *C.L. Kane and E.J. Mele*, ***PRL*** *95, 226801 (2005)*
Theo2: *B.A. Bernevig et al.*, ***Science*** *314, 1757 (2006)*
Exp: *M. König et al.*, ***Science*** *318, 766 (2007)*

Theo: *L. Fu, et al.*, ***PRL*** *98, 106803 (2007)*
Exp1: *Zhang H. et al.*, ***Nat. Phys.*** *5, 438 (2009)*
Exp3: *S. Takafumi et al.*, ***PRL*** *105, 136802 (2010)*

3D

S/TI proximity effect





J. Shen et al. (2013)

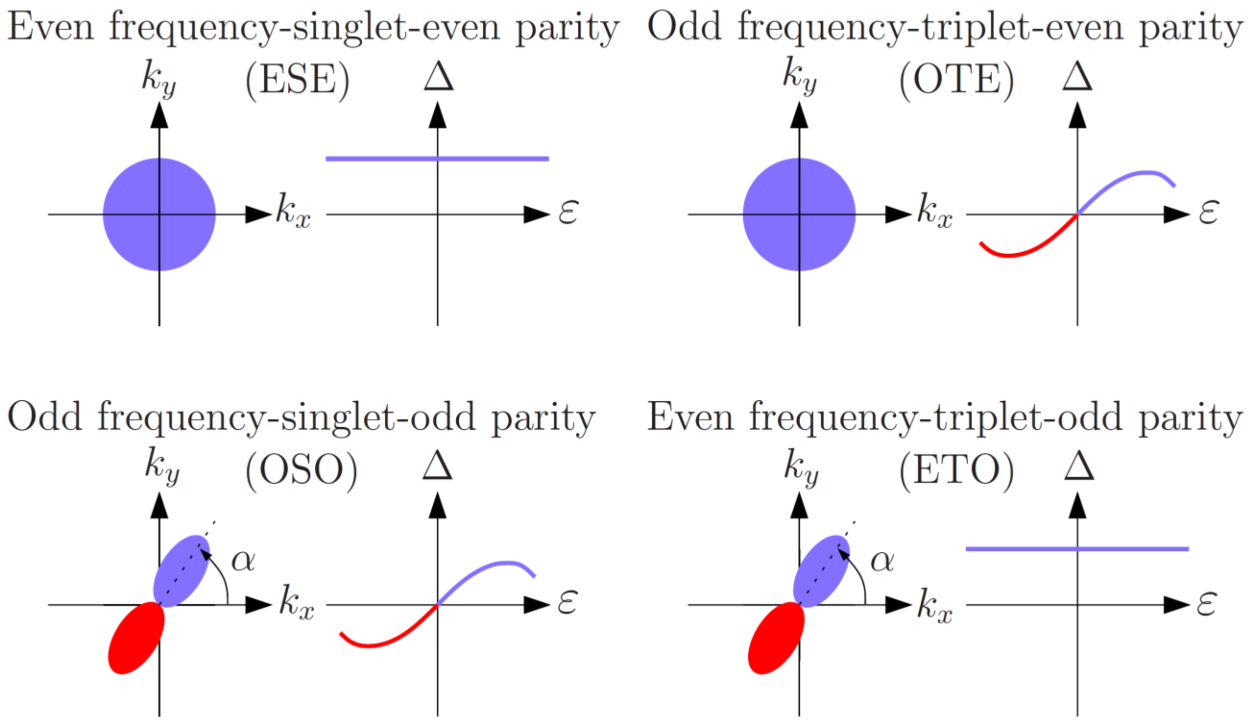
Symmetry classification of induced pair potential

Fermi-Dirac statistics

Symmetry of pair wave functions:

$$\mathbf{k} \otimes \sigma \otimes \omega = \mathrm{odd}$$

	$F_{\sigma\sigma'}(\omega,k) = -F_{\sigma'\sigma}(-\omega,-k)$		
	$\omega \to -\omega$	$\sigma \leftrightarrow \sigma'$	$k \to -k$
ESE	$+$	$-$	$+$
OSO	$-$	$-$	$-$
ETO	$+$	$+$	$-$
OTE	$-$	$+$	$+$



J. Linder et al., PRB (2008)

Majorana fermion

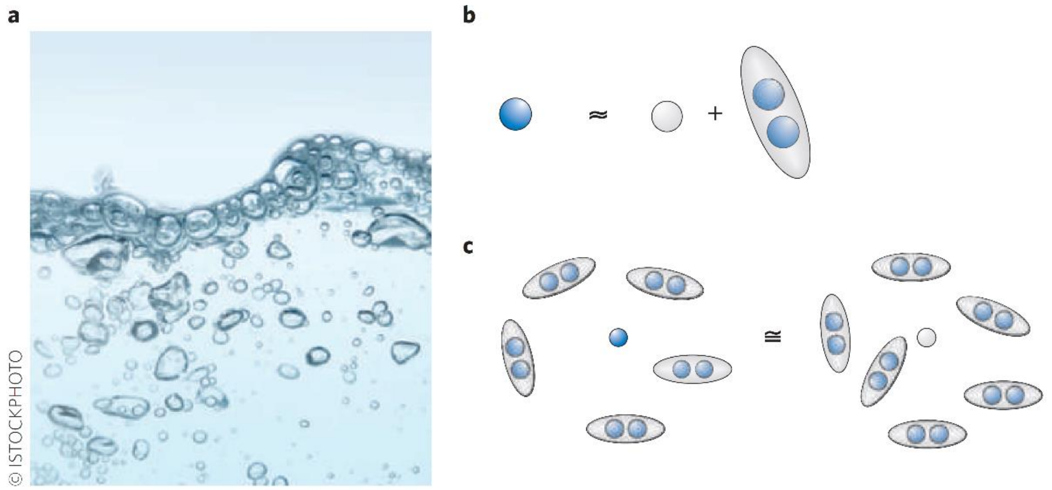
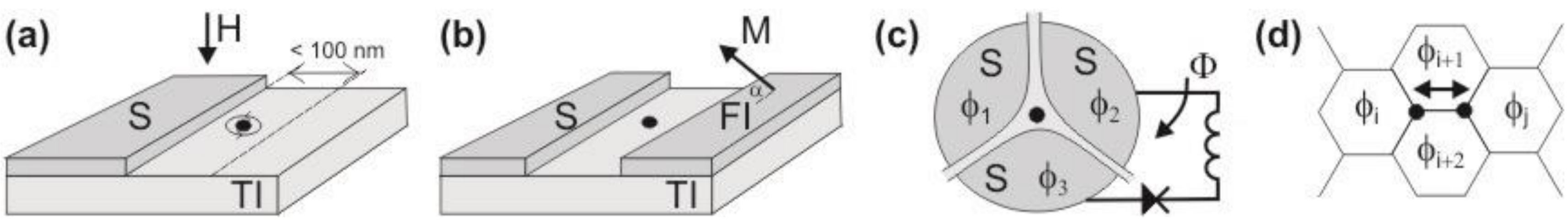


Figure 1 | Antimatter matters in the solid state. **a**, A familiar concept in solid-state physics, holes are bubbles of missing electrons in the Fermi sea of the electronic spectrum, behaving like positively charged electrons. **b**, In a superconductor, the properties of electrons (blue) and holes (grey) are drastically modified by their interaction with the surrounding sea of Cooper pairs; a hole can attract or bind to a Cooper pair, and acquire negative charge. **c**, More importantly, Cooper pairs cluster around holes and thin out around electrons, in such a way that no rigorous distinction between them remains.

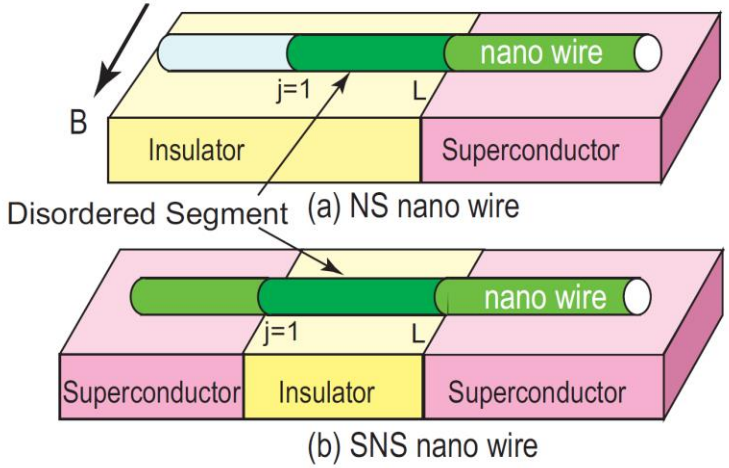
F. Wilczek, Nat. Phys. (2009)



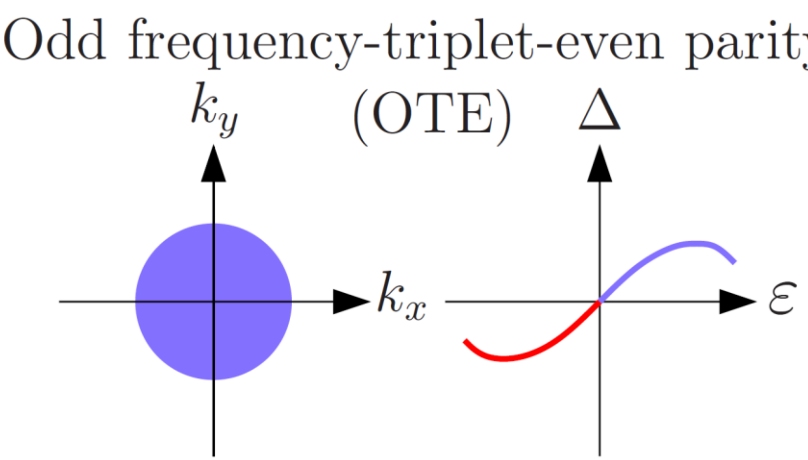
Fu, Kane, PRL (2008)

Tanaka, Yokoyama, Nagaosa, PRL (2009)

Odd-frequency and Majorana



Asano, Tanaka, PRB (2013)



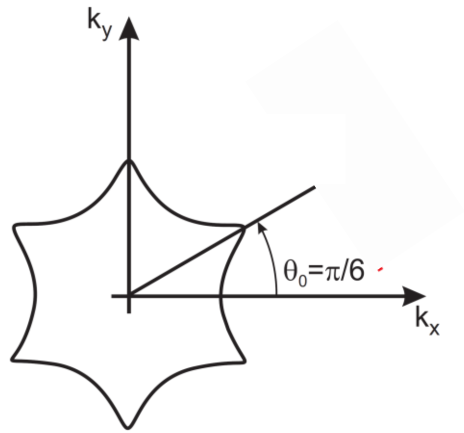
The physics behind the anomalous transport can be understood in terms of the odd-frequency Cooper pairing. We conclude that Majorana fermions and odd-frequency Cooper pairs in solids are two sides of a same coin.

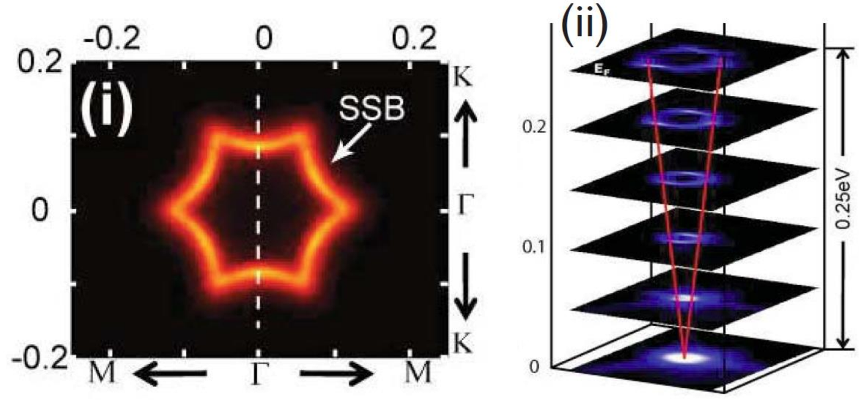
Hexagonal warping in 3D topological insulators

$$\hat{H}(\mathbf{k}) = -\mu + v(k_x\hat{\sigma}_y - k_y\hat{\sigma}_x) + \hat{H}_{\rm w}(\mathbf{k})$$

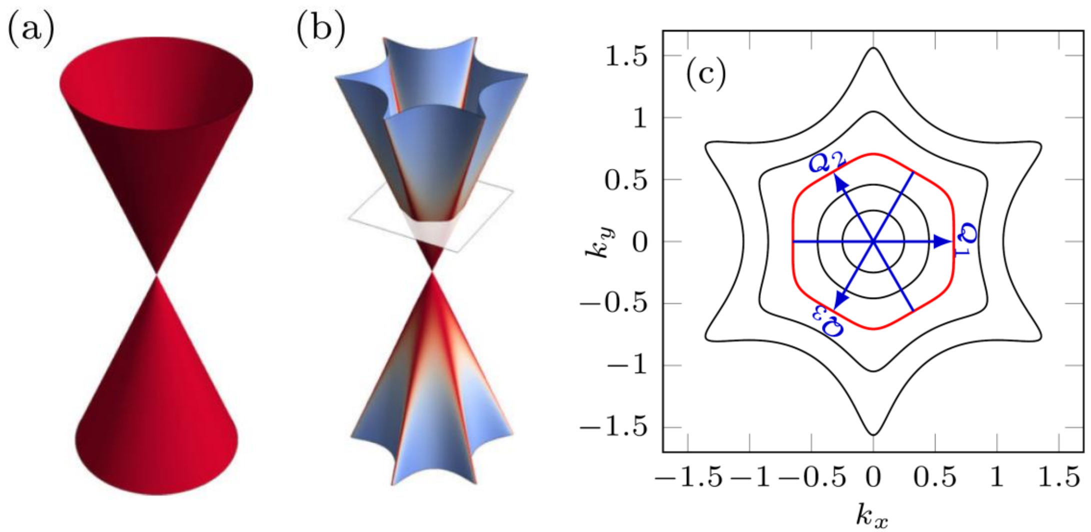
$$\hat{H}_{\rm w}(\mathbf{k}) = \lambda k^3 \cos(3\theta)\hat{\sigma}_z$$

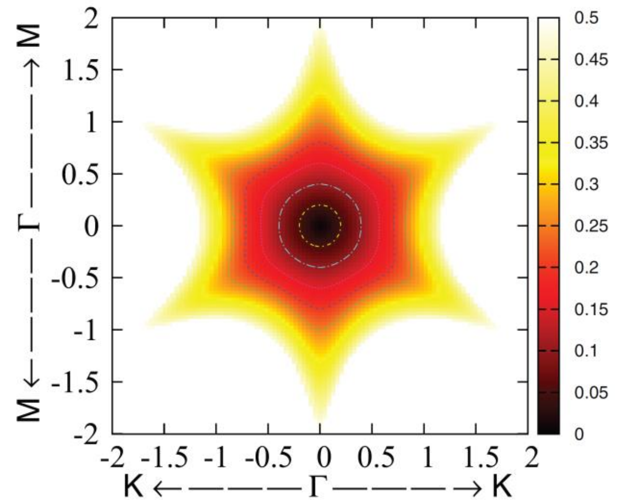
Fu, PRL (2009)





Chen et al., Science (2009)





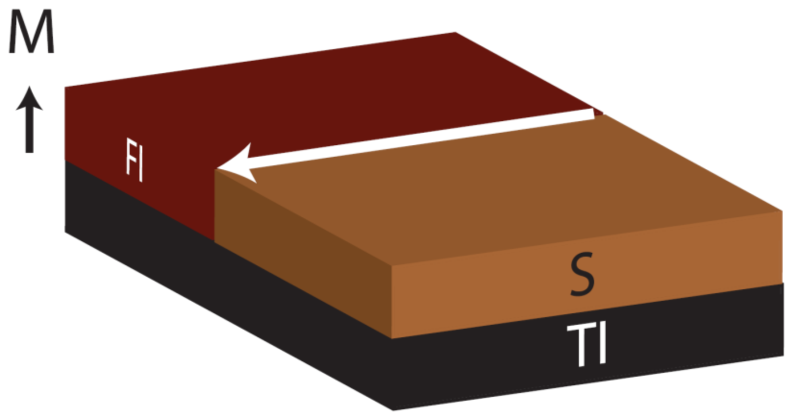
Mendle, Kotetes, Schon, PRB (2015)

Li, Carbotte, PRB (2013)

Model: S/FI hybrid on TI surface

Bogoliubov – de Gennes – Dirac Hamiltonain

$$\check{H}_S(\mathbf{k}) = \begin{pmatrix} \hat{H}(\mathbf{k}) + M\hat{\sigma}_z & \hat{\Delta} \\ -\hat{\Delta} & -\hat{H}^*(-\mathbf{k}) - M\hat{\sigma}_z \end{pmatrix}$$



Green's function (Nambu + spin space)

$$\left[E - \check{H}_S(\mathbf{k})\right]\check{G} = \check{1} \qquad \check{G} = \begin{pmatrix} \hat{G}_{\mathrm{ee}} & \hat{G}_{\mathrm{eh}} \\ \hat{G}_{\mathrm{he}} & \hat{G}_{\mathrm{hh}} \end{pmatrix}$$

Anomalous Green's function

Bergeret, Volkov, Efetov, RMP (2005)

$$\hat{G}_{\mathrm{eh}} = i\left(f_0\hat{\sigma}_0 + f_x\hat{\sigma}_x + f_y\hat{\sigma}_y + f_z\hat{\sigma}_z\right)\hat{\tau}_y$$

$(\uparrow\downarrow - \downarrow\uparrow)$ singlet — triplet $(\uparrow\uparrow - \downarrow\downarrow)$ — triplet $(\uparrow\uparrow + \downarrow\downarrow)$ — triplet $(\uparrow\downarrow + \downarrow\uparrow)$

No warping

$$\check{H}_S(\mathbf{k}) = \begin{pmatrix} \hat{H}(\mathbf{k}) + M\hat{\sigma}_z & \hat{\Delta} \\ -\hat{\Delta} & -\hat{H}^*(-\mathbf{k}) - M\hat{\sigma}_z \end{pmatrix}$$

$$\hat{H}(\mathbf{k}) = -\mu + v(k_x\hat{\sigma}_y - k_y\hat{\sigma}_x)$$

$$\hat{G}_{\mathrm{eh}} = i\left(f_0\hat{\sigma}_0 + f_x\hat{\sigma}_x + f_y\hat{\sigma}_y + f_z\hat{\sigma}_z\right)\hat{\tau}_y$$

Anomalous Green's function symmetry, Z is even in E and k

$$f_0 = \frac{\Delta}{Z}\left(E^2 + M^2 - \mu^2 - \Delta^2 - v^2k^2\right), \quad (\uparrow\downarrow - \downarrow\uparrow) \quad \textbf{ESE}$$

$$f_x = \frac{2\Delta}{Z}kv\left[\mu\sin(\theta) + iM\cos(\theta)\right], \quad (\uparrow\uparrow - \downarrow\downarrow) \quad \textbf{ETO}$$

$$f_y = -\frac{2\Delta}{Z}kv\left[\mu\cos(\theta) - iM\sin(\theta)\right], \quad (\uparrow\uparrow + \downarrow\downarrow) \quad \textbf{ETO}$$

$$f_z = \frac{2\Delta}{Z}EM. \quad (\uparrow\downarrow + \downarrow\uparrow) \quad \textbf{OTE}$$

No warping

$$\check{H}_S(\mathbf{k}) = \begin{pmatrix} \hat{H}(\mathbf{k}) + M\hat{\sigma}_z & \hat{\Delta} \\ -\hat{\Delta} & -\hat{H}^*(-\mathbf{k}) - M\hat{\sigma}_z \end{pmatrix}$$

$$\hat{H}(\mathbf{k}) = -\mu + v(k_x\hat{\sigma}_y - k_y\hat{\sigma}_x)$$

$$\hat{G}_{\text{eh}} = i\left(f_0\hat{\sigma}_0 + f_x\hat{\sigma}_x + f_y\hat{\sigma}_y + f_z\hat{\sigma}_z\right)\hat{\tau}_y$$

Anomalous Green’s function symmetry, Z is even in E and k

$$f_0 = \frac{\Delta}{Z}\left(E^2 + M^2 - \mu^2 - \Delta^2 - v^2k^2\right), \quad (\uparrow\downarrow - \downarrow\uparrow)$$ **ESE**

$$f_x = \frac{2\Delta}{Z}kv\left[\mu\sin(\theta) + iM\cos(\theta)\right], \quad (\uparrow\downarrow + \downarrow\uparrow)$$ **ETO**

$$f_y = -\frac{2\Delta}{Z}kv\left[\mu\cos(\theta) - iM\sin(\theta)\right], \quad (\uparrow\uparrow + \downarrow\downarrow)$$ **ETO**

$$f_z = \frac{2\Delta}{Z}EM. \quad (\uparrow\downarrow + \downarrow\uparrow)$$ **OTE**

Finite warping

$$\check{H}_S(\mathbf{k}) = \begin{pmatrix} \hat{H}(\mathbf{k}) + M\hat{\sigma}_z & \hat{\Delta} \\ -\hat{\Delta} & -\hat{H}^*(-\mathbf{k}) - M\hat{\sigma}_z \end{pmatrix}$$

$$\hat{H}(\mathbf{k}) = -\mu + v(k_x\hat{\sigma}_y - k_y\hat{\sigma}_x) + \hat{H}_{\mathrm{w}}(\mathbf{k})$$

$$\hat{G}_{\mathrm{eh}} = i\left(f_0\hat{\sigma}_0 + f_x\hat{\sigma}_x + f_y\hat{\sigma}_y + f_z\hat{\sigma}_z\right)\hat{\tau}_y$$

ESE **OTE**
OSO **ETO**

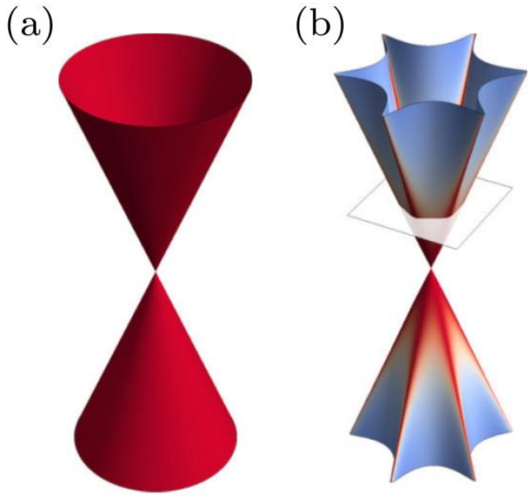
Anomalous Green's function z-component $(\uparrow\downarrow + \downarrow\uparrow)$

$$f_z = f_z^- + f_z^+,$$

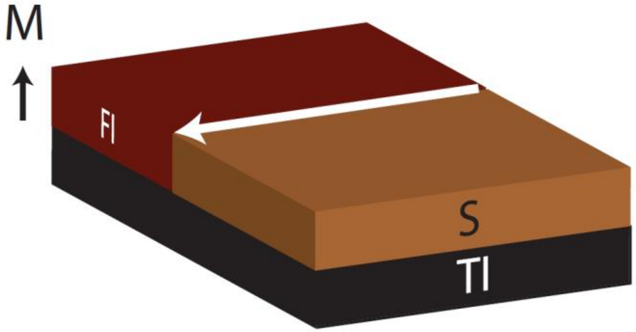
$$f_z^- = EMF_{\mathrm{even}} - \mu\lambda k^3\cos(3\theta)F_{\mathrm{odd}},$$ **OTE**

$$f_z^+ = EMF_{\mathrm{odd}} - \mu\lambda k^3\cos(3\theta)F_{\mathrm{even}}.$$ **ETO**

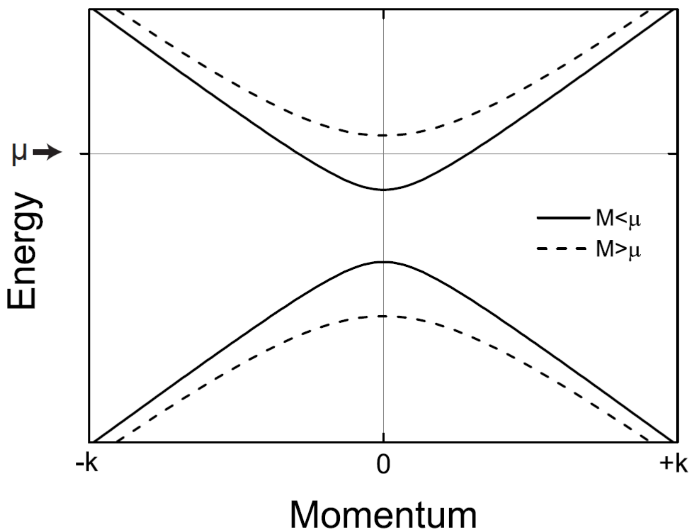
$$\theta_n = \pi/6 + \pi n/3$$

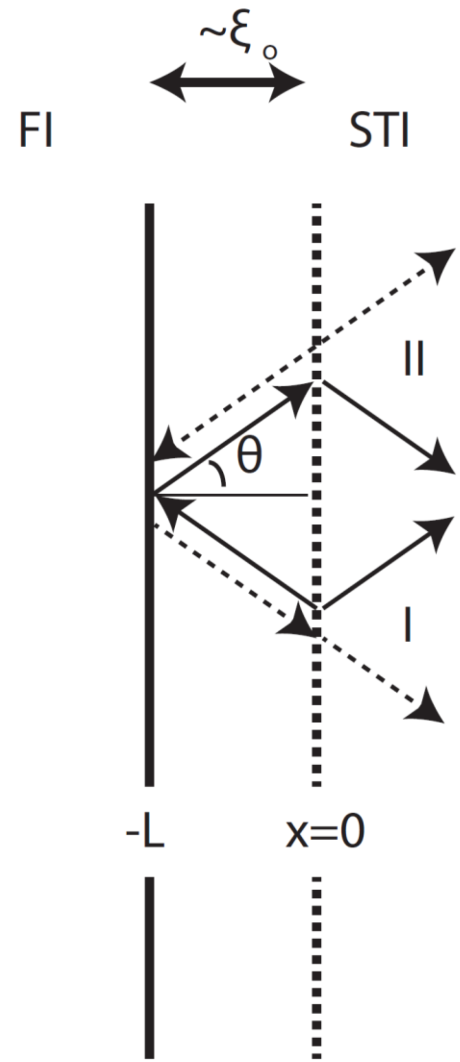


Majorana fermion realization



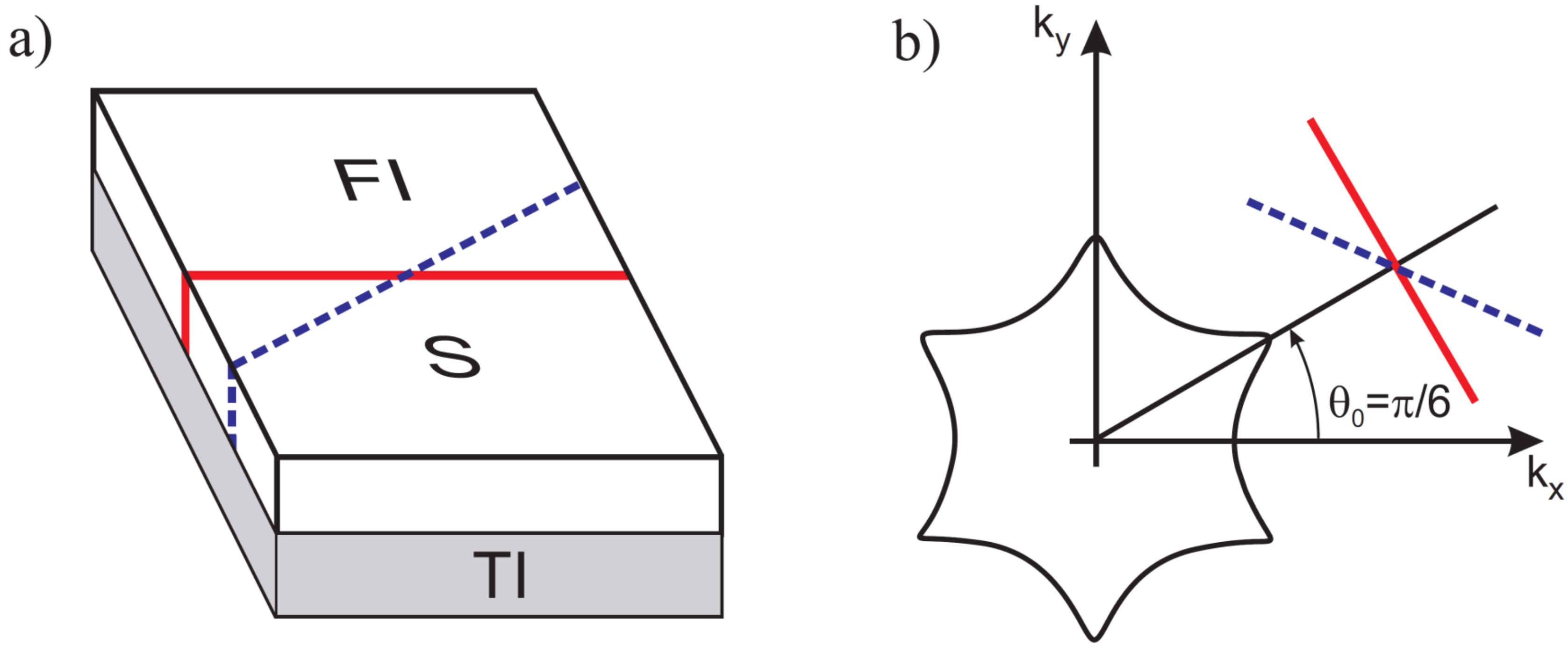
$$\frac{E}{\Delta} = -\sin(\theta)$$
$$= -k_y/|k|.$$





Snelder, Golubov, Asano, Brinkman (2015)

Majorana fermion and warping

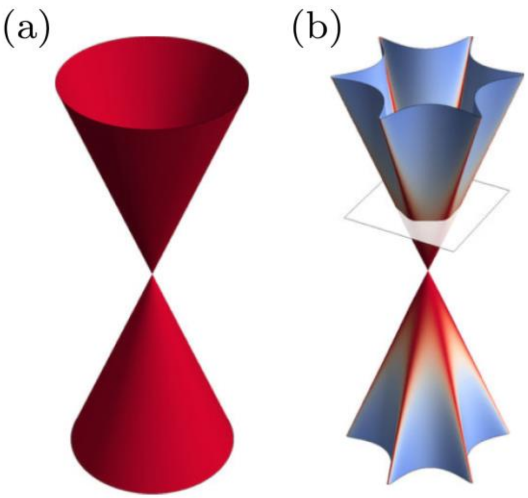


Vasenko, Golubov, Silkin, Chulkov, arXiv:1606.00905 (2016)

Spontaneous currents

$$\hat{H}_M(\mathbf{k}) = -\mu + v(k_x\hat{\sigma}_y - k_y\hat{\sigma}_x) + \lambda k^3\cos(3\theta)\hat{\sigma}_z + M\hat{\sigma}_z$$

$$\boxed{\hat{H}(\mathbf{k}) = -\mu + vk_x\hat{\sigma}_y + \hat{\sigma}_x M_x + \hat{\sigma}_y M_y + \hat{\sigma}_z M_z}$$



Nesterov, Houzet, Meyer, arXiv:1512.03042 (2016)

Review

- We discuss singlet to triplet mixing in proximized 3D topological insulators with warped surface state
- We establish the selection rule for Majorana Fermion realization in S/FI structures formed on the surface of the TI: S/FI boundary should be properly aligned with respect to the snowflake contour.
- Spontaneous currents in S/TI hybrids.

Thank you!